Azonosító jel: | | | | | | | | | | | | | | |

ÉRETTSÉGI VIZSGA • 2023. május 9.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

minden vizsgázó számára

2023. május 9. 9:00

Időtartam: 300 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

OKTATÁSI HIVATAL

Instructions to candidates

1. The time allowed for this examination paper is 300 minutes. When that time is up, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below**. *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!

5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**

6. **Make sure that calculations of intermediate results are also possible to follow.**

7. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**

8. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.

9. Always state the final result (the answer to the question of the problem) in words, too!

10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

12. Please, **do not write in the grey rectangles**.

I.

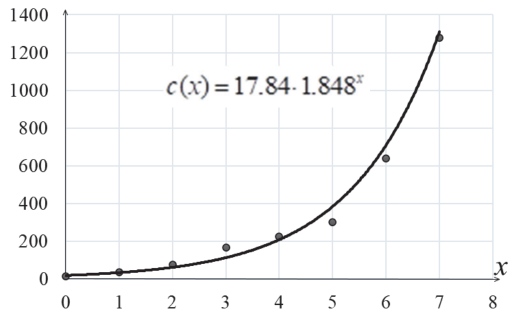
1. The table below, found on the internet[1], shows the total power output (in megawatts) of all solar-to-electrical energy converter devices in Hungary over the last few years.

year	2012	2013	2014	2015	2016	2017	2018	2019
total power output (MW)	12	35	77	168	225	300	640	1277

a) Data in the table above was used to create another table (below) that shows how many times greater the total power output grew each year, as compared to that of the previous year. Fill in the three missing cells and calculate the mean and standard deviation of the seven numbers.

year	2013	2014	2015	2016	2017	2018	2019
The total power output in the given year is this many times greater **than it was in the previous year:**	2.92		2.18		1.33		2.00

The data from the first table was entered into a spreadsheet program and, assuming the total power output follows an exponential growth pattern, a best-fit curve was calculated. The program gave the equation of the best-fit curve as $c(x) = 17.84 \cdot 1.848^x$ where x is the number of years since 2012 (x is a natural number), and $c(x)$ is the expected total power output given by this model.



b) For the year 2018, calculate the percentage difference of the model's prediction from the actual value of 640 MW.

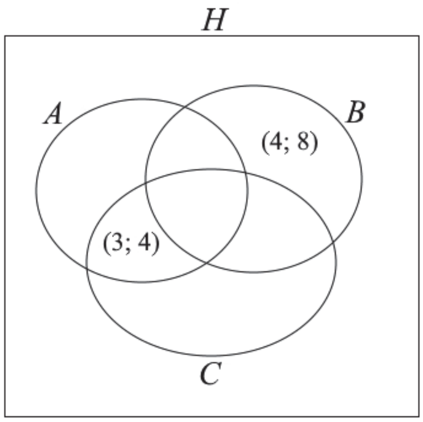
c) Solve the equation $17.84 \cdot 1.848^x = 40000$ in the set of real numbers.

a)	4 points	
b)	3 points	
c)	4 points	
Ö.:	11 points	

[1] https://hu.wikipedia.org/wiki/Napenergia_Magyarországon

2. Let the fundamental set H be the set of pairs of positive integers. Sets A, B and C are all subsets of H:
$A = \{(a; b) \mid a$ and b are relatively primes (co-primes)$\}$;
$B = \{(a; b) \mid a$ is a divisor of $b\}$;
$C = \{(a; b) \mid$ at least one of the numbers a and b is a prime$\}$.
(If $a \neq b$ then the pairs $(a; b)$ and $(b; a)$ are considered different.)

a) Appropriate pairs have already been entered into two regions of the Venn diagram below. Write one appropriate pair into each of the remaining six regions of the diagram.



Consider the following two statements (a, b and c are positive integers):
I. If c is a divisor of ab then c is a divisor of a or c is a divisor of b.
II. If a is a divisor of c and b is a divisor of c then ab is a divisor of c.

b) Determine the truth value (true or false) of each statement. Explain your answer.

c) State the converse of statement I and determine the logical value of the converse. If the converse is true, prove it. If the converse is false, give a counterexample.

a)	6 points	
b)	4 points	
c)	4 points	
T.:	14 points	

3. The vertical cross section of a straight, horizontal tunnel is an 8.1 m tall circular segment that is part of a circle of 6-metre radius. The length of the tunnel is 340 m. (The image is illustrative only.)

a) Show that the central angle subtended by the arc of this circular segment is 221° (rounded to the nearest degree).

b) Calculate the volume of the tunnel. Round your answer to the nearest thousand m^3.

The curved inner surface of the tunnel is covered with ceramic tiles.

c) Calculate the tiled area in m^2-s.

a)	4 points	
b)	7 points	
c)	3 points	
T.:	14 points	

4. A farm is selling size L (large) and size M (medium) eggs. They ask 10 forints more for each large egg than they ask for each medium egg. A storekeeper bought 450 eggs from this farm last week and paid a total 25 800 forints. This week he also bought 450 eggs but paid only 23 700 forints, because this week he bought as many size M eggs as he bought size L last week (this, of course, also means that this week the number of size L eggs bought is the same as that of last week's size M-s).

a) What does one size M egg cost, what is the price of one size L egg and how many size M eggs did the shopkeeper buy last week? (The price of eggs did not change.)

Balázs is making scrambled eggs for himself using 4 eggs. There are 6 eggs in the fridge, 5 of them good and 1 that went bad, however, Balázs does not know this. Balázs is in a hurry, he is breaking the eggs one after the other into a bowl. If he happens to pick 4 good eggs, he goes and scrambles them right away. If, however, he puts the bad one into the bowl after 2 or 3 good ones, he will not get his scrambled eggs. (Balázs realises immediately if a bad egg goes into the bowl and junks the whole thing then and there. However, if he still has 4 or more eggs left at this point, he starts again.)

b) Calculate the probability that Balázs can make his scrambled eggs with 4 eggs.

a)	7 points	
b)	5 points	
T.:	12 points	

II.

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

5. In the cyclic quadrilateral *ABCD* it is known that $AB = 15$, $BC = 10$. Diagonal *BD* divides the angle *ABC* such that $ABD\sphericalangle = 20°$, $DBC\sphericalangle = 40°$.

a) Prove that the length of diagonal *AC* is exactly $5\cdot\sqrt{7}$!

b) Prove that the angles of triangle *ACD* are 20°, 40° and 120°.

c) Calculate the area of quadrilateral *ABCD*.

In kite *KLMN* there are right angles at vertices *K* and *M*, diagonal *KM* is 9.6 cm long. Diagonal *LN* is a line of symmetry that is divided into two sections by the point of intersection of the two diagonal such that the difference between the lengths of the sections is 2.8 cm.

d) Calculate the area of the kite.

a)	3 points	
b)	2 points	
c)	4 points	
d)	7 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

6. **a)** Three girls and four boys are going to the cinema. They hold tickets to seven adjacent seats in the same row. How many different seating arrangements are there for them to sit, given that no two girls may sit next to one another?

b) There are only 3 unoccupied seats left in the first row. Right behind these, in the second row, there are also 3 unoccupied seats left. How many different seating arrangements are there for a party of six people if those sitting in the second row must each be taller than those sitting directly in front of them? (All six people are of different heights.)

c) A simple graph has 8 vertices and 13 edges. The degree of one of the vertices is 6. Prove that this graph must contain a circuit of 3 vertices (i.e. a "triangle" of edges).

a)	6 points	
b)	6 points	
c)	4 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

7. While developing a new car, engineers try to determine the fuel consumption at various speeds. They have three data points so far: at 40 km/h the fuel consumption is 9.6 litres on 100 km, at 70 km/h it is 6.9 litres and at 120 km/h it is 6.4 litres.

a) Use the above data to calculate the average fuel consumption on 100 km for the following journey: the car is first driven at 40 km/h for 30 minutes, then it is driven at 120 km/h for 50 minutes.

Three engineers are trying to fit a best estimate function *f* for the fuel consumption over the given data points. A best estimate function is such that the sum $|f(40)-9.6|+|f(70)-6.9|+|f(120)-6.4|$ is as small as possible.

Engineer Csaba uses the linear function $f_1(x)=11.2-0.04x$, while Engineer Dóra suggests an absolute value function: $f_2(x)=\frac{|x-100|}{10}+4$ to best estimate the fuel consumption of the car on 100 km (here *x* is the speed of the car in km/h and the fuel consumption is given in litres).

b) Based on the three given data points which of the functions f_1 and f_2 is a better estimate for the fuel consumption, **as outlined above**?

Engineer Elemér is looking for the quadratic function $f_3(x)=ax^2+bx+c$ that is perfectly fitting all three data points, i.e. $f_3(40)=9.6$; $f_3(70)=6.9$ and $f_3(120)=6.4$.

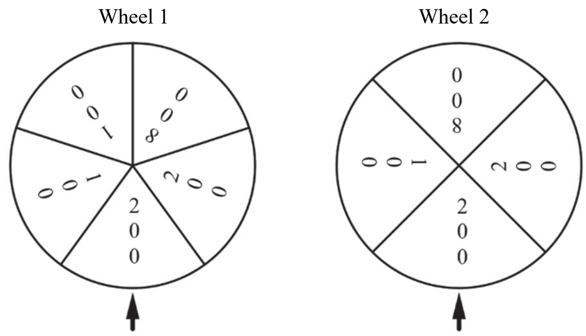
c) Calculate the value of parameters *a*, *b* and *c*.

a)	4 points	
b)	5 points	
c)	7 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

8. Visitors of a summer festival can try their luck at a game of "Wheels of fortune". The player has to turn two different wheels of fortune which will then stop randomly at one or the other number. (Sectors on the same wheel each have the same central angle and each of the nine sectors show one of the numbers 100, 200 or 800.)

Before turning the wheels, players must pay an entry fee of 200 forints. If the two wheels stop such that they show the same number the player will **win** as many forints as is the sum of the numbers shown. (E.g. if both wheels stop at 200, as shown in the diagram here, then the player wins 200+200 = 400 forints.) If the wheels stop at different numbers, the player does not win anything.



a) Calculate the probability that Wheel 1 will stop at 100 exactly 4 times out of 10 games.

When the player turns the two wheels their **gain** is the difference of what they win and the entry fee.

b) What is the average expected gain on one game?

When the sum of the two numbers shown is 1000, it is called "bingo". If a player turns a "bingo" they may select a song to be played in the festival tent.

c) Prove that the probability of a "bingo" is 0.2.

d) How many times must the wheels be turned so that the probability of at least one "bingo" is at least 95%?

a)	3 points	
b)	5 points	
c)	3 points	
d)	5 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9.
Write the number of the problem NOT selected in the blank square on page 2.

9. The function f is defined over the set of real numbers. The formula for the **derivative function** of f is: $f'(x) = (x-2)^2 \cdot (x-5)$.

a) Give the type and location of all local extremes of function f.

b) Give the formula for function f, such that its graph goes through the point (0; 1).

c) Prove that the function $g: \mathbf{R} \to \mathbf{R};\ g(x) = \dfrac{3x^3 + x}{x^2 + 1}$ is strictly monotone increasing.

a)	5 points	
b)	5 points	
c)	6 points	
T.:	16 points	

	Number of problem	score			
		maximum	awarded	maximum	awarded
Part I	1.	11		**51**	
	2.	14			
	3.	14			
	4.	12			
Part II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date

examiner

	pontszáma **egész számra** kerekítve	
	elért	programba beírt
I. rész		
II. rész		

dátum

dátum

javító tanár

jegyző